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PROJECTIVE PRODUCT OF SEMIRINGS

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ABSTRACT

In this paper we authors investigated some results on various derivations in the projective product of semirings.We have proved that every pair of derivations/semiderivations/generalized derivations/generalized semiderivations/Jordan derivations/generalized Jordan derivations/Jordan semiderivation/Generalized Jordan semiderivation on a pair of semirings give rise to respective derivations on the projective product of the semirings. The converse results are also studied fruitfully. The similar result can be investigated in the case of the projective product of n number of semirings.
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I. INTRODUCTION AND PRELIMINARIES

The notion of semiring was introduced by H.S.Vandiver [8] in 1934. The structure of semiderivations of prime rings has been studied by C.L. Chuang [7] .Jonathan.S.Golan introduced the definition of derivations in semirings in 1969. J.Bergen introduced the notion of semiderivatons in prime rings [6] in the year 1983. The research work of classical ring theory to semiring theory has been drawn interest of many prominent Mathematicians over the globe to determine many basic properties of semirings and to enrich the field of research in algebra.In this paper we authors investigated some results on various derivations in the projective product of semirings.

DEFINITION 1.1. A semiring S is a nonempty set S Equipped with two binary operations + and .such that

1. (S, +) is a commutative monoid with identity element 0
2. (S, .)is a monoid with identity element 1
3. Multiplication left and right distributes over addition.

DEFINITION 1.2. A semiring S is said to be prime if $xsy = 0$ implies $x = 0$ or $y = 0$ for all $x, y \in S$

DEFINITION 1.3. A semiring S is said to be semiprime if $xsx = 0$ implies $x = 0$ for all $x \in S$

DEFINITION 1.4. An additive mapping $d:S \rightarrow S$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in S$

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DEFINITION 1.5. An additive mapping D:S→ S is called a generalized derivation if D(xy) = D(x)y + xd(y) holds for all x, y ∈ S .

DEFINITION 1.6. An additive mapping f:S→ S is called a semiderivation associated with a function g : S → S if for all x, y ∈ S
(i)f (xy) = f (x)g(y) + xf (y) = f (x)y + g(x)f (y), (ii)f (g(x)) = g(f (x))

DEFINITION 1.7. An additive mapping F:S→ S is called a generalized semiderivation associated with a semiderivation f : S → S if for all x, y ∈ S
(i)F (xy) = F (x)g(y) + xf (y) = F (x)y + g(x)f (y), (ii)F (g(x)) = g(F (x)) .

If g = I i.e., an identity mapping of S then all semiderivations associated with g are merely ordinary derivations. If g is any endomorphism of S, then semiderivations are of the form f (x) = x − g(x) .

DEFINITION 1.8. An additive mapping J:S→ S is called a Jordan derivation if for all x ∈ S, J (x^2) = J (x)x + xJ (x) .

DEFINITION 1.9. An additive mappingG:S→ Sis called a generalized

Jordan derivation if there exists a Jordan derivation J and for all x ∈ S
$G(x^2)$ =G(x)x + xJ (x) .

DEFINITION 1.10.An additive mappingh:S→ Sis called a Jordan
semiderivation associated with a function g : S → S if for all x ∈ S
(i)$h(x^2)$ = h(x)g(x) + xh(x) = h(x)x + g(x)h(x), (ii)h(g(x)) = g(h(x)) .

DEFINITION 1.11. An additive mapping H: S → Sis called a generalized

Jordan semiderivation associated with a Jordan semiderivation h: S →S if for all lx∈S (i)H (x^2) = H (x)g(x) + xh(x) = H (x)x + g(x)h(x), (ii)H (g(x)) = g(H (x)) .

DEFINITION 1.12. Let S_1 and S_2 be two semirings. Let S = S_1X S_2 . We define addition and multiplication on S by $(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$ and $(x_1, x_2).(y_1, y_2) = (x_1.y_1, x_2.y_2)$ for all $x = (x_1, x_2) \in S$ where $x_1 \in S_1$ and $x_2 \in S_2$. With respect to this addition and multiplication S is a semiring. We call this semiring as the projective product of semirings.

II. RESULTS

THEOREM 2.1.Let S_1 and S_2 be two semirings and S be their projective product. If d_1 and d_2 are derivations on S_1 and S_2respectively then d : S → S defined by $d(x) = d(x_1,x_2) = (d_1(x_1),d_2(x_2))$ is a derivation on S for all $x = (x_1,x_2) \in S$ where $x_1 \in S_1$ and $x_2 \in S_2$respectively

Proof: Let $x = (x_1, x_2)$, $y = (y_1, y_2) \in S$.

Then clearly d iswell defined and additive.

Now, $d(xy) = d(x_1y_1, x_2y_2)$

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= $(d_1(x_1y_1), d_2(x_2y_2))$
= $(d_1(x_1)y_1 + x_1d_1(y_1), d_2(x_2)y_2 + x_2d_2(y_2))$
= $(d_1(x_1)y_1, d_2(x_2)y_2) + (x_1d_1(y_1), x_2d_2(y_2))$
= $(d_1(x_1), d_2(x_2)(y_1, y_2) + (x_1, x_2)(d_1(y_1, d_2(y_2))$
= $d(x)y + xd(y)$ for all $x, y \in S$

Hence d is a derivation on S

THEOREM 2.2. Let S_1 and S_2 be two semirings and S be their projective product. If D_1 and D_2 are generalized derivations on S_1 and S_2respectively then $D : S \to S$ defined by $D(x) = D(x_1, x_2) = (D_1(x_1), D_2(x_2))$ is a generalized derivation on S for all $x = (x_1, x_2) \in S$ where $x_1 \in S_1$ and $x_2 \in S_2$.

Proof: Let $x = (x_1, x_2)$, $y = (y_1, y_2) \in S$.

Then clearly D iswell defined and additive.

Now, $D(xy) = D(x_1y_1, x_2y_2)$

= $(D_1(x_1y_1), D_2(x_2y_2))$
= $(D_1(x_1)y_1 + x_1d_1(y_1), D_2(x_2)y_2 + x_2d_2(y_2))$
= $(D_1(x_1)y_1, D_2(x_2)y_2) + (x_1d_1(y_1), x_2d_2(y_2))$
= $(D_1(x_1), D_2(x_2)(y_1, y_2) + (x_1, x_2)(d_1(y_1), d_2(y_2))$
= $D(x)y + xd(y)$ for all $x, y \in S$

Hence D is a generelaized derivation on S

THEOREM 2.3. Let S_1 and S_2 be two semirings and S be their projective product. Two semiderivations f_1 and f_2 associated with functions g_1 and g_2 on S_1 and S_2respectivelygive rise to a semiderivation f associated with the function g. on S defined by $f(x) = f(x_1, x_2) = (f_1(x_1), f_2(x_2))$ for all $x = (x_1, x_2) \in S$ where $x_1 \in S_1$ and $x_2 \in S_2$respectievely.

Proof: Let $x = (x_1, x_2)$, $y = (y_1, y_2) \in S$.

Clearly f and g arewelldefined and additive.

Now, $f(xy) = f(x_1y_1, x_2y_2)$
= $(f_1(x_1y_1), f_2(x_2y_2))$
= $(f_1(x_1)g_1(y_1) + x_1f_1(y_1), f_2(x_2)g_2(y_2) + x_2f_2(y_2))$
= $(f_1(x_1)g_1(y_1), f_2(x_2)g_2(y_2) + (x_1f_1(y_1), x_2f_2(y_2))$
= $(f_1(x_1), f_2(x_2)(g_1(y_1), g_2(y_2)) + (x_1, x_2)(f_1(y_1), f_2(y_2))$
= $f(x)g(y) + xf(y)$ for all $x, y \in S$ (1)
= $f(g_1(x_1), g_2(x_2))$
= $(f_1(g_1(x_1)), f_2(g_2(x_2)))$
= $(g_1(f_1(x_1)), g_2(f_2(x_2)))$
= $g(f_1(x_1, f_2(x_2)) = g(f(x))$ (3)

Thus from(1),(2)and(3) f is a semiderivation associated with the function g on S .

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THEOREM 2.4. Let S_1 and S_2 be two semirings and S be their projective product. Two generalized semiderivations F_1 and F_2 associated with semiderivations f_1 and f_2 on S_1 and S_2respectievely give rise to a generalizedsemiderivation F associated with the semiderivation f on S defined by F (x) = F (x1, x2) = (F1(x1), F2(x2)) for all x = (x1, x2) $\in$ S where x1$\in$ S1 and x2$\in$ S2respectively.

Proof: Let $x = (x_1, x_2)$, $y = (y_1, y_2) \in S$.

Clearly F is welldefined and additive.

Now, $F(xy) = F(x_1y_1, x_2y_2)$

$= (F_1(x_1y_1), F_2(x_2y_2))$

$= (F_1(x_1)g_1(y_1) + x_1f_1(y_1), F_2(x_2)g_2(y_2) + x_2f_2(y_2))$

$= (F_1(x_1)g_1(y_1), F_2(x_2)g_2(y_2)) + (x_1f_1(y_1), x_2f_2(y_2))$

$= (F_1(x_1), F_2(x_2)(g_1(y_1), g_2(y_2)) + (x_1, x_2)(f_1(y_1), f_2(y_2))$

$= F(x)g(y) + xf(y)$ for all $x, y \in S$ (1)

$= F(g_1(x_1), g_2(x_2))$

$= (F_1(g_1(x_1)), F_2(g_2(x_2)))$

$= (g_1(F_1(x_1)), g_2(F_2(x_2)))$

$= g(F_1(x_1, F_2(x_2)) = g(F(x))$ (3)

Thus from (1),(2)and(3) F is a generelaizedsemiderivation associated with the semiderivation f on S .

THEOREM 2.5. Let S_1 and S_2 be two semirings and S be their projective product. Two Jordan derivations J_1 and J_2 on S_1 and S_2respectively give rise to a Jordan derivation J on S defined by $J(x) = J(x_1, x_2) = (J_1(x_1), J_2(x_2))$ for all $x = (x_1, x_2) \in S$ where $x_1 \in S_1$ and $x_2 \in S_2$respectively.

Proof: Let $x = (x_1, x_2) \in S$.

Then clearly J iswelldefined and additive.

Now, $J(x^2) = J(x^2, x^2)$

$= (J_1(x^2{}_1), J_2(x^2{}_2))$

$= (J_1(x_1)x_1 + x_1J_1(x_1), J_2(x_2)x_2 + x_2J_2(x_2))$

$= (J_1(x_1)x_1, J_2(x_2)x_2) + (x_1J_1(x_1), x_2J_2(x_2))$

$= (J_1(x_1), J_2(x_2)(x_1, x_2) + (x_1, x_2)(J_1(x_1), J_2(x_2))$

$= J(x)x + xJ(x)$ for all $x \in S$

Hence J is a jordan derivation on S

THEOREM 2.6. Let S_1 and S_2 be two semirings and S be their projective product. If G_1 and G_2 are generalized Jordan derivations on S_1 and S_2respectively, then $G : S \to S$ defined by $G(x) = G(x_1, x_2) = (G_1(x_1), G_2(x_2))$ is a generalized Jordan derivation on S for all $x = (x_1, x_2) \in S$ where $x_1 \in S_1$ and $x_2 \in S_2$.

Proof: Let $x = (x_1, x_2) \in S$.

Then clearly G iswelldefined and additive.

Now, $G(x^2) = G(x^2{}_1, x^2{}_2)$
= $(G_1(x^2{}_1), G_2(x^2{}_2))$
= $(G_1(x_1)x_1 + x_1J_1(x_1), G_2(x_2)x_2 + x_2J_2(x_2))$
= $(G_1(x_1)x_1, G_2(x_2)x_2) + (x_1J_1(x_1), x_2J_2(x_2))$
= $(G_1(x_1), G_2(x_2)(x_1, x_2) + (x_1, x_2)(J_1(x_1), J_2(x_2))$
= $G(x)x + xJ(x)$ for all $x \in S$

Hence G is a generelaized Jordan derivation associated with the Jordan derivation J on S .

THEOREM 2.7. Let S_1 and S_2 be two semirings and S be their projective product. If h_1 and h_2 are Jordan semiderivations on S_1 and S_2respectively, then $h : S \rightarrow S$ defined by $h(x) = h(x_1, x_2) = (h_1(x_1), h_2(x_2))$ is a Jordan semiderivation on S for all $x = (x_1, x_2) \in S$ where $x_1 \in S_1$ and $x_2 \in S_2$.

Proof: Let $x = (x_1, x_2) \in S$.

Then clearly h iswelldefined and additive.

Now, $h(x^2) = h(x^2{}_1, x^2{}_2)$
= $(h_1(x^2{}_1), h_2(x^2{}_2))$
= $(h_1(x_1)g_1(x_1) + x_1h_1(x_1), h_2(x_2)g_2(x_2) + x_2h_2(x_2))$
= $(h_1(x_1)g_1(x_1), h_2(x_2)g_2(x_2)) + (x_1h_1(x_1), x_2h_2(x_2))$
= $(h_1(x_1), h_2(x_2)(g_1(x_1), g_2(x_2)) + (x_1, x_2)(h_1(x_1), h_2(x_2))$
= $h(x)g(x) + xh(x)$ for all $x \in S$ (1)

Similarly we can prove $h(xy) = h(x)x + g(x)h(x)$ for all $x \in S$ (2)

Also, $h(g(x)) = h(g(x_1, x_2))$ for all $x \in S$
= $h(g_1(x_1), g_2(x_2))$
= $(h_1(g_1(x_1)), h_2(g_2(x_2)))$
= $(g_1(h_1(x_1)), g_2(h_2(x_2)))$
= $g(h_1(x_1), h_2(x_2)) = g(h(x))$ for all $x \in S$ (3)

Hence from (1), (2) and (3) h is a Jordan semiderivation on S .

THEOREM 2.8. Let S_1 and S_2 be two semirings and S be their projective product. If H_1 and H_2 are generalized Jordan semiderivations on S_1 and S_2respectively, then $H : S \rightarrow S$ defined by $H(x) = H(x_1, x_2) = (H_1(x_1), H_2(x_2))$ is a generalized Jordan semiderivation on S for all $x = (x_1, x_2) \in S$ where $x_1 \in S_1$ and $x_2 \in S_2$.

Proof: Let $x = (x_1, x_2) \in S$.

Then clearly H iswelldefined and additive.

Now, $H(x^2) = H(x^2{}_1, x^2{}_2)$
= $(H_1(x^2{}_1), H_2(x^2{}_2))$
= $(H_1(x_1)g_1(x_1) + x_1h_1(x_1), H_2(x_2)g_2(x_2) + x_2h_2(x_2))$
= $(H_1(x_1)g_1(x_1), H_2(x_2)g_2(x_2)) + (x_1h_1(x_1), x_2h_2(x_2))$

= $(H_1(x_1), H_2(x_2)(g_1(x_1), g_2(x_2)) + (x_1, x_2)(h_1(x_1, h_2(x_2))$

= $H(x)g(x) + xh(x)$ for all $x \in S$ (1)

Similarly we can prove $H(xy) = H(x)x + g(x)h(x)$ for all $x \in S$ (2)

Also, $H(g(x)) = H(g(x_1, x_2))$ for all $x \in S$

= $H(g_1(x_1), g_2(x_2))$

= $(H_1(g_1(x_1)), H_2(g_2(x_2)))$

= $(g_1(H_1(x_1)), g_2(H_2(x_2)))$

= $g(H_1(x_1), H_2(x_2)) = g(H(x))$ for all $x \in S$ (3)

Hence from (1), (2) and (3) H is a generelaized Jordan semiderivation associated with the Jordan semiderivation h on S .

THEOREM 2.9. For every derivation/ generalized derivation/ semiderivation/ generalized semiderivation/ Jordan derivation/ Jordan generalized derivation/ Jordan semiderivation/ Jordan generalized semiderivation d on the projective product $S = S_1 X S_2$ there exists corresponding derivations/ generalized derivations/ semiderivations/ generalized semi derivations/ Jordan derivations/ Jordan generalized derivations/ Jordan semiderivations/ Jordan generalized semiderivations d_1 and d_2 on the semirings S_1 and S_2respectively.

Proof: Let d be a derivation on the projective product $S = S_1 X S_2$ defined by $d(x) = d(x_1, x_2) = (d_1(x_1), d_2(x_2)) = (s_1, s_2)$ where $d_1 : S_1 \rightarrow S_1$ and $d_2 : S_2 \rightarrow S_2$ are defined by $d_1(x_1) = s_1 = f(d(x_1, x_2))$ and $d_2(x_2) = s_2 = h(d(x_1, x_2))$ respectively for all $x_1 \in S_1$ and $x_2 \in S_2$. Now for all $x_1, y_1 \in S_1$

$d_1(x_1 + y_1) = f(d(x_1 + y_1, x_2 + y_2))$

= $f(d(x_1, x_2) + d(y_1, y_2))$

= $d_1(x_1) + d_2(x_2)$

= $f(d(x)y + xd(y))$

= $f((d_1(x_1), d_2(x_2))(y_1, y_2) + (x_1, x_2)(d_1(y_1), d_2(y_2)))$

= $f((d_1(x_1)y_1, d_2(x_2)y_2)) + f((x_1 d_1(y_1), x_2 d_2(y_2)))$

= $d_1(x_1)y_1 + x_1 d_1(y_1)$

Hence d_1 is a derivation on S_1 .

Similarly we can prove d_2 is a derivation on S_2 .

Proceeding as above we can prove the other results also.

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